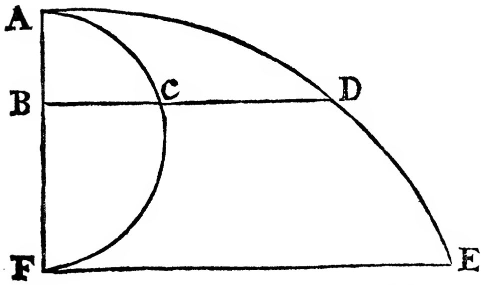
IV. *De Figurarum Geometricè irrationalium Quadraturis.* *Autore* Johanne Craig.



SIT ACF Semicirculus, cujus Diameter eſt AF, ADE Curva Geometricè irrationalis, cujus Ordinatim applicata BD ſecat ſemicirculum in C. Quantitates verò ſic deſignentur; Diameter $AF = 2a$, Abſciſſa $AB = y$, Arcus $AC = v$, Ordinata $BD = z$: ſitque $Z = rvy^n$ æquatio generalis exprimens naturas Curvarum Geometricè irrationalium ADE, in qua r denotat quantitatem quamlibet datam & determinatam, & n exponentem indefinitum quantitatis indeterminatæ y. Dico Aream.

$$ABD = \frac{rvy^{n+1}}{n+1} - qv + \sqrt{2ay - yy} \times \frac{ra}{\overline{n+1}|^2} y^n + \frac{2nra^2 + ra^2}{n \times \overline{n+1}|^2} y^{n-1} +$$

$$\frac{aA \times \overline{2n-1}}{n-1} y^{n-2} + \frac{aB \times \overline{2n-3}}{n-2} y^{n-3} + \frac{aC \times \overline{2n-5}}{n-3} y^{n-4} + \frac{aD \times \overline{2n-7}}{n-4} y^{n-5} +$$

$$\frac{aE \times \overline{2n-9}}{n-5} y^{n-6} + \&c.$$

De hac Serie Infinitâ hæc ſunt notanda: (1.) Quod Literæ majuſculæ A, B, C, D, E, &c. deſignent coefficientes terminorum ipſis immediatè præcedentium, ſciz: $A = \frac{2nraa + raa}{n \times \overline{n+1} \times \overline{n+1}}$ $B = \frac{aA \times \overline{2n-1}}{n-1}$, $C = \frac{aB \times \overline{2n-3}}{n-2}$ & ſic porro. (2.) Quod ſi exponens n ſit numerus integer & poſitivus, aut nihilo æqualis, vel etiam ſi 2 n ſit numerus impar, tum Quadratura Spatii ABD exhibeatur per Quantitatem finitam; ſerie in his caſibus abrumpente. (3.) Quod q deſignet terminum ultimo abrumpentem. (4.) Quod omnes illæ Figuræ in quibus ſeries abrumpitur habeant unam portionem Geometricè Quadrabilem, ex ipſâ ſerie facillimè aſſignabilem: Nimirum ſi capiatur

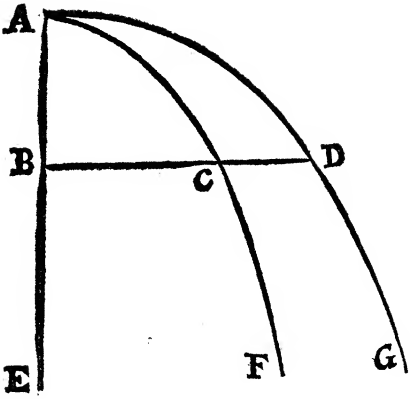
atur Abſciſſa $y=\overline{r}|^{-\frac{1}{n+1}} \times \overline{nq+q}|^{\frac{1}{n+1}}$; Erit huic Abſciſſæ competens Area Geometricè quadrabilis. (5.) Quod ſolus terminus irrationalis $\sqrt{2ay-yy}$ in terminos ipſum ſequentes ſit multiplicandus.

Exemplum 1. Sit $z=v$, quia in hoc caſu $r=1$, $n=0$, ideo $\frac{ra}{\overline{n+1}|^{2}}y^{n}$ eſt terminus ultimò abrumpens; quare $q=a$, unde $ABD=vy-av+a\sqrt{2ay-y2}$: & proinde ſi (per not. 4) capiatur Abſciſſa $y=a$, id eſt, ſi ordinata tranſeat per circuli centrum erit portio huic competens Geometricè Quadrabilis, ſcil. Area$=aa$, id eſt, Radii Quadrato.

Exemp. 2. Sit $z=\frac{vy}{a}$, quia in hoc caſu $r=\frac{1}{a}$, $n=1$, ideo $\frac{2nraa+raa}{n\times\overline{n+1}|^{2}}y^{n-1}$ eſt terminus ultimo abrumpens, quare $q=\frac{3a}{4}$; unde $ABD=\frac{vy^2}{2a}-\frac{3av}{4}+\frac{\overline{y+2a}\sqrt{2ay-y^2}}{4}$ & proinde ſi (per not. 4) capiatur $y=\sqrt{\frac{3aa}{2}}$, erit huic abſciſſæ competens Area Geometricè Quadrabilis, Scil. Area $=\sqrt{:\sqrt{6a^4}-\frac{3a^2}{2}} \times \sqrt{\frac{3a^2}{32}}+\frac{3a}{4}$.

Exemplum 3. Sit $z=\frac{vy^2}{aa}$; In hoc caſu $r=\frac{1}{aa}$, $n=2$, ideo $\frac{aA\times\overline{2n-1}}{n-1}y^{n-2}$ eſt terminus ultimò abrumpens, ergo $q=\frac{5a}{6}$; unde per Seriem Infinitam erit

$ABD=\frac{6vy^3-15a^3v+\overline{2ay^2+5a^2y+15a^3}\sqrt{2ay-yy}}{18a^2}$ Et proinde ſi (per not. 4) Capiatur $y=\sqrt[3]{\frac{5a^3}{2}}$, erit abſciſſæ competens Area Geometrice quadrabilis: ſcil. Area$=\frac{2ay^2+5a^2y+15a^2}{18a}\times\sqrt{2ay--yy}$



Secundò. Sit A C F Parabola, cujus Axis A E, Vertex A, & latus rectum (B a). Sitque A D G Curva Geometricè irrationalis, cujus Ordinatim applicata B D ſecat Parabolam in C. Et vocetur Abſciſſa $AB=y$, Ordinata $BD=z$, Arcus Parabolicus $AC=v$. Sitque æquatio generalis exprimens Naturas infinitarum Curvarum irrationalium, hæc. $Z=rvy^{n}$ in qua r denotat quantitatem datam & determinatam, & n exponentem indefinitum quantitatis indeterminatæ y. Dico Aream

$$ABD = \frac{ry^{n+1}xv}{n+1} - qv + \sqrt{2ay+yy}\,x - \frac{r}{n+2\times n+1}y^{n+1} - \frac{ra}{n+2\times\overline{n+1}|^{2}}y^{n} + \frac{raa\times 2n+1}{n\times n+2\times\overline{n+1}|^{2}}y^{n-1} - \frac{aA\times 2n-1}{n-1}y^{n-2} + \frac{aB\times 2n-3}{n-2}y^{n-3} - \frac{aC\times 1n-5}{n-3}y^{n-4} + \&c.$$

De hac ſerie hæc ſunt notanda: (1.) Quod literæ majuſculæ, A, B, C, &c. denotent coefficientes terminorum ipſis præcedentium. (2.) Quod ſi exponens n ſit integer poſitivus aut nihilo æqualis, aut etiam ſi 2 n ſit numerus impar, tum Quadratura exhibeatur per numerum Terminorum finitum; ſerie in his caſibus abrumpente. (3.) Quod $+q$ ſit æqualis termino ultimo abrumpenti. (4.) Quod ex terminis quantitatem $\sqrt{2ay+yy}$ multiplicantes ultimò abrumpens ſit duplicandus. (5.) Quod omnes illæ figuræ, in quibus n eſt numerus integer poſitivus & impar, vel generalius, omnes illæ Figuræ, in quibus ultimus terminus abrumpens habet ſignum affirmativum ſeu +, habeant unam portionem Geometricè Quadrabilem, & ex ipſa ſerie facilè aſſignabilem, ſumendo abſciſſam ut in not. 4. præcedentis Seriei.

Exemplum 1. Sit $z=v$, quia in hoc caſu $r=1$, $n=0$, ideo terminus ultimò abrumpens eſt $-\frac{ra}{n+2\times\overline{n+1}|^{2}}y^{n}$, unde $+q=-\frac{a}{2}$ (per not. 3) & quia in hoc caſu $-\frac{a}{2}$ eſt terminus ultimo

ultimo abrumpens, ideo — a eſt ultimus terminus in $\sqrt{2ay+yy}$ multiplicandus (per not. 4). Adeoque

$$ABD = vy + \frac{av}{2} + \sqrt{2ay+y^2}\,x\,\overline{-\frac{1}{2}y - a}.$$

Exemp. 2. Sit $z = \frac{vy}{a}$, quia in hoc caſu $r = \frac{1}{a}$, $n = 1$, ideo terminus ultimò abrumpens eſt $\frac{raa \times \overline{2n+1}}{n \times \overline{n+2} \times \overline{n+1}|^2}\, y^{n-1} = \frac{a}{4}$, unde $q = \frac{a}{4}$ & $\frac{a}{2}$ ultimus terminus in $\sqrt{2ay+yy}$ multiplicandus; adeoque $ABD = \frac{vyy}{2a} - \frac{av}{4} + \sqrt{2ay+yy}\,x - \frac{y^2}{6a} - \frac{y}{12} + \frac{a}{2}$ Et ſi capiatur $y = \sqrt{\frac{aa}{2}}$, erit Area competens huic abſciſſæ Geometricè Quadrabilis, ſcil. $\text{Area} = \frac{1}{12}\sqrt{:\sqrt{2a^4} + \frac{a^2}{2}} \times 5a - \sqrt{\frac{a^2}{2}}:$

Plura habeo hujuſmodi Theoremata, pro Figuris ex circulo Parabolâ & dependentibus; ſed hæc duo, ſpeciminis gratiâ, ſufficiant ad oſtendendum uſum Methodi meæ in tractatu noſtro de Quadraturis editæ, in determinandis Figurarum irrationalium Quadraturis, ad quas nulla alia (quantùm ſcio) Methodus hactenus porrigitur.

V. *Part of a Letter of Mr.* Robert Tredwey, *to Dr.* Leonard Plukenet, *Dated* Jamaica, Feb. 12. 169$^{6}_{7}$. *giving an Account of a great piece of Ambergrieſe thrown on that Iſland; with the Opinion of ſome there about the way of its Production.*

I Shall only at preſent let you know the Account I received from *Ambergrieſe Ben*, for ſo the Man is called from the vaſt Quantity of that valuable Commodit